Int. J. Multiphase Flow, Vol. 5, pp. 159–164
Pergamon/Elsevier, 1979. Printed in Great Britain

BRIEF COMMUNICATION

ON THE FORM OF THE PRESSURE TERMS IN THE MOMENTUM AND ENERGY EQUATIONS OF TWO-PHASE FLOW MODELS

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(*Received* 20 *October* 1978)

1. INTRODUCTION

For several years now, there has been a controversy, as recalled by Sha & Soo (1979) on the form of the pressure term in the phasic *momentum* equations of two-phase flow models.

As already pointed out (e.g. Delhaye 1977) there should not be any controversy, since both forms are acceptable, *provided* the other terms in the equations are correctly written. The origin of the controversy is very simple indeed and, since this controversy is time—and effort—consuming, a new attempt to close it is deemed worthwhile. Such is the main purpose of this note.

Surprisingly enough, there does not appear to be any controversy on the form of the pressure term in the phasic *energy* equations, but since the problem in analogous to the momentum equation problem, it is also considered hereunder.

2. THE TWO FORMS OF THE MOMENTUM EQUATION FOR ONE PHASE

The *local-instantaneous* linear momentum balance may be written:

$$\frac{\partial \rho_K \mathbf{V}_K}{\partial t} + \nabla.(\rho_K \mathbf{V}_K \mathbf{V}_K) - \nabla.\ \bar{\bar{T}}_K - \rho_K \mathbf{g} = 0, \qquad [1]$$

the subscript K referring to the phase which is present at the particular point and instant t under consideration, ρ being the density, $\mathbf{V}$ the velocity, $\bar{\bar{T}}$ the (total) stress tensor and $\mathbf{g}$ the acceleration of gravity (or, more generally, the external force per unit of mass). The product $\mathbf{V}_K\mathbf{V}_K$ is the tensor product.

Introducing in [1] the pressure P through the expression

$$\bar{\bar{T}}_K = -P_K \bar{\bar{I}} + \bar{\bar{\tau}}_K, \qquad [2]$$

where $\bar{\bar{I}}$ is the unit tensor and $\bar{\bar{\tau}}$ the deviatoric stress tensor, results in:

$$\frac{\partial \rho_K \mathbf{V}_K}{\partial t} + \nabla.(\rho_K \mathbf{V}_K \mathbf{V}_K) + \nabla P_K - \nabla.\bar{\bar{\tau}}_K - \rho_K \mathbf{g} = 0, \qquad [3]$$

which is strictly equivalent to [1].

The balance equations which are involved in the controversy are averaged equations. They are space-averaged (the forms obtained through either volume- or area- or segment-averaging being similar) and time-averaged (or statistically averaged, the forms obtained through either time-averaging or statistical averaging being similar). The two kinds of averaging operators are commutative (Delhaye & Achard 1976, 1977). Therefore it is possible, without loss of generality, to deal hereunder with equations which are first, say, volume-averaged then time-averaged. More specifically, the volume-averaging is performed hereunder, as a typical example, over the volume $\mathcal{V}_K(z, t)$ occupied by the phase K between two fixed cross-sections located at $z-(\Delta z/2)$ and $z+(\Delta z/2)$ within a pipe, z being the abscissa along the pipe axis (It is a simple matter to apply the same procedure to the other cases. Consideration of the above volume is at the basis of most current models for pipe flows and in particular, when Δz is taken as vanishingly small, at the basis of most one-dimensional models).

The volume $\mathcal{V}_K$ is part of the volume $\mathcal{V}(z, t)$ limited by the pipe wall and the two foregoing cross-sections. The volumetric concentration of phase K at time t within volume $\mathcal{V}$ (void fraction or its complement to 1) is $\alpha_K(z, t) = \mathcal{V}_K/\mathcal{V}$. The volume $\mathcal{V}_K$ is limited by a surface which may consist of three portions:

A portion located in the cross-section planes. This portion does not usually contain interfaces, but it may in particular cases.

A portion $C_K(z, t)$ located on the pipe wall.

A portion $I(z, t)$ consisting of interfaces.

Assuming the quantities involved are sufficiently regular, transformation of the volume-averaged equations derived from [1] or [3] into practical equations requires (Delhaye & Achard 1976, 1977):

a. Application of the Leibniz rule to transform

$$\int_{\mathcal{V}_K(z,t)} \frac{\partial \mathbf{f}_K}{\partial t} \, \mathrm{d}\mathcal{V}$$

($\mathbf{f}_K$ being any vectorial—or scalar—function of space and time defined for phase K) into the time derivative of some function. It yields:

$$\int_{\mathcal{V}_K(z,t)} \frac{\partial \mathbf{f}_k}{\partial t} \, \mathrm{d}\mathcal{V} = \frac{\partial}{\partial t} \int_{\mathcal{V}_K(z,t)} \mathbf{f}_K \, \mathrm{d}\mathcal{V} - \int_{I(z,t)} \mathbf{f}_K(\mathbf{V}_I \cdot \mathbf{n}_K) \, \mathrm{d}\Sigma - \int_{C_K(z,t)} \mathbf{f}_K(\mathbf{V}_c \cdot \mathbf{n}_K) \, \mathrm{d}\Sigma, \quad [4]$$

where $\mathbf{n}_K$ is the unit vector of the outside normal to I or C_K, and $\mathbf{V}_I \cdot \mathbf{n}_K$ and $\mathbf{V}_C \cdot \mathbf{n}_K$ are respectively the speeds of displacement of surfaces I and C_K. The last term of [4] is usually omitted, since it is zero whenever the pipe wall is fixed with respect to the frame of reference.

b. Application of the Gauss theorem to transform

$$\int_{\mathcal{V}_K(z,t)} \nabla \cdot \bar{\bar{M}}_K \, \mathrm{d}\mathcal{V}$$

($\bar{\bar{M}}_K$ being any symmetric tensor—or vector—function of space and time defined for phase K) into the derivative with respect to z of some function. It yields

$$\int_{\mathcal{V}_K(z,t)} \nabla \cdot \bar{\bar{M}}_K \, \mathrm{d}\mathcal{V} = \frac{\partial}{\partial z} \int_{\mathcal{V}_K(z,t)} \bar{\bar{M}}_K \cdot \mathbf{n} \, \mathrm{d}\mathcal{V} + \int_{I(z,t)} \bar{\bar{M}}_K \cdot \mathbf{n}_K \, \mathrm{d}\Sigma + \int_{C_K(z,t)} \bar{\bar{M}}_K \cdot \mathbf{n}_K \, \mathrm{d}\Sigma, \quad [5]$$

where $\mathbf{n}$ is the unit vector of the $0z$ axis.

Introducing the definition of the volume-average $\langle Q_K \rangle$ of any quantity Q_K over $\mathcal{V}_K$

$$\int_{\mathcal{V}_K} Q_K \, \mathrm{d}\mathcal{V} \stackrel{\Delta}{=} \mathcal{V}_K \langle Q_K \rangle = \mathcal{V} \alpha_K \langle Q_K \rangle$$

and using $(\rho \mathbf{V}\mathbf{V}) \cdot \mathbf{n} = \rho(\mathbf{V} \cdot \mathbf{n})\mathbf{V}$, enables the instantaneous volume-averaged linear momentum equation to be written either:

$$\begin{aligned}&\frac{\partial}{\partial t}(\mathscr{V}\alpha_K\langle\rho_K\mathbf{V}_K\rangle)+\frac{\partial}{\partial z}(\mathscr{V}\alpha_K\langle\rho_K W_K\mathbf{V}_K\rangle)+\frac{\partial}{\partial z}(\mathscr{V}\alpha_K\langle P_K\rangle\mathbf{n})\\&-\frac{\partial}{\partial z}(\mathscr{V}\alpha_K\langle\bar{\bar{\tau}}_K\cdot\mathbf{n}\rangle)+\int_I[\rho_K(\mathbf{V}_K-\mathbf{V}_I)\cdot\mathbf{n}_K]\mathbf{V}_K\,\mathrm{d}\Sigma+\int_I P_K\mathbf{n}_K\,\mathrm{d}\Sigma\\&-\int_I(\bar{\bar{\tau}}_K\cdot\mathbf{n}_K)\,\mathrm{d}\Sigma+\int_{C_K}[\rho_K(\mathbf{V}_K-\mathbf{V}_C)\cdot\mathbf{n}_K]\mathbf{V}_K\,\mathrm{d}\Sigma+\int_{C_K}P_K\mathbf{n}_K\,\mathrm{d}\Sigma\\&-\int_{C_K}(\bar{\bar{\tau}}_K\cdot\mathbf{n}_K)\,\mathrm{d}\Sigma-\mathscr{V}\alpha_K\langle\rho_K\rangle\mathbf{g}=0\end{aligned}\qquad[6]$$

(starting from [1], with $W_K=\mathbf{V}_K\cdot\mathbf{n}$), or

$$\begin{aligned}&\frac{\partial}{\partial t}(\mathscr{V}\alpha_K\langle\rho_K\mathbf{V}_K\rangle)+\frac{\partial}{\partial z}(\mathscr{V}\alpha_K\langle\rho_K W_K\mathbf{V}_K\rangle)+\mathscr{V}\alpha_K\langle\nabla P_K\rangle\\&-\frac{\partial}{\partial z}(\mathscr{V}\alpha_K\langle\bar{\bar{\tau}}_K\cdot\mathbf{n}\rangle)+\int_I[\rho_K(\mathbf{V}_K-\mathbf{V}_I)\cdot\mathbf{n}_K]\mathbf{V}_K\,\mathrm{d}\Sigma\\&-\int_I(\bar{\bar{\tau}}_K\cdot\mathbf{n}_K)\,\mathrm{d}\Sigma+\int_{C_K}[\rho_K(\mathbf{V}_K-\mathbf{V}_C)\cdot\mathbf{n}_K]\mathbf{V}_K\,\mathrm{d}\Sigma\\&-\int_{C_K}(\bar{\bar{\tau}}_K\cdot\mathbf{n}_K)\,\mathrm{d}\Sigma-\mathscr{V}\alpha_K\langle\rho_K\rangle\mathbf{g}=0\end{aligned}\qquad[7]$$

(starting from [3]).

The first wall term is usually omitted in both of these equations, since it is zero whenever there is no mass transfer through the pipe wall. Also, the last differential term is usually neglected.

Applying the Leibniz theorem yields, for any quantity Q which is a continuous function of time and with a constant time interval Δt:

$$\frac{\partial}{\partial t}\int_{t-(\Delta t/2)}^{t+(\Delta t/2)}Q\,\mathrm{d}t=Q\left(t+\frac{\Delta t}{2}\right)-Q\left(t-\frac{\Delta t}{2}\right),$$

whence

$$\int_{t-(\alpha t/2)}^{t+(\Delta t/2)}\frac{\partial Q}{\partial t}\,\mathrm{d}t=Q\left(t+\frac{\Delta t}{2}\right)-Q\left(t-\frac{\Delta t}{2}\right)=\frac{\partial}{\partial t}\int_{t-(\Delta t/2)}^{t+(\Delta t/2)}Q\,\mathrm{d}t.$$

Therefore, time averaging [6] or [7] leaves the equations formally unchanged, except for the multiplication of every term by Δt and adjunction of the time averaging symbol, according to

$$\int_{t-(\Delta t/2)}^{t+(\Delta t/2)}Q\,\mathrm{d}t\stackrel{\Delta}{=}\bar{Q}\Delta t$$

Equations [6] and [7] are two forms of the momentum equation which are *strictly equivalent*. This can be easily verified by using [5] once more, with $\bar{\bar{M}}_K=P_K\bar{\bar{I}}$. The result is

$$\int_{\mathscr{V}_K}\nabla P_K\,\mathrm{d}\mathscr{V}=\frac{\partial}{\partial z}\int_{\mathscr{V}_K}P_K\mathbf{n}\,\mathrm{d}\mathscr{V}+\int_I P_K\mathbf{n}_K\,\mathrm{d}\Sigma+\int_{C_K}P_K\mathbf{n}_K\,\mathrm{d}\Sigma$$

i.e.

$$\underbrace{\mathscr{V}\alpha_K\langle\nabla P_K\rangle}_{\text{term of [7]}} = \underbrace{\frac{\partial}{\partial z}(\mathscr{V}\alpha_K\langle P_K\rangle\mathbf{n}) + \int_I P_K\mathbf{n}_K\,\mathrm{d}\Sigma + \int_{C_K} P_K\mathbf{n}_K\,\mathrm{d}\Sigma}_{\text{terms of [6].}} \qquad [8]$$

3. DISCUSSION

As stated in the introduction, either [6] or [7] may be used, provided the interfacial and wall pressure terms are *not* omitted in [6] and provided they are modelled in a way which is *consistent* with the rest of the model.

This point is important, since assumptions have to be made, in particular on the way to express the averaged quantities in terms of the main dependent variables. For example, in most one-dimensional models (in which the equations are obtained through division of [6] or [7] by Δz, Δz being made vanishingly small), the terms containing the pressure are evaluated, assuming that any nonuniformity of P_K within $\mathscr{V}_K$ and during the time interval Δt may be neglected.

In this case, $\mathscr{A}$ being the cross-section area ($\mathscr{V} = \mathscr{A}\Delta z$), [8]) becomes, after division by Δz

$$\mathscr{A}\alpha_K\frac{\partial P_K}{\partial z}\mathbf{n} = \frac{\partial}{\partial z}(\mathscr{A}\alpha_K P_K\mathbf{n}) + P_K \lim_{(\Delta z\to 0)}\frac{1}{\Delta z}\left[\int_I \mathbf{n}_K\,\mathrm{d}\Sigma + \int_{C_K}\mathbf{n}_K\,\mathrm{d}\Sigma\right],$$

which shows that, in [6], after division by Δz,

$$\lim_{(\Delta z\to 0)}\frac{1}{\Delta z}\left[\int_I P_K\mathbf{n}_K\,\mathrm{d}\Sigma + \int_{C_K} P_K\mathbf{n}_K\,\mathrm{d}\Sigma\right] = -P_K\frac{\partial(\mathscr{A}\alpha_K)}{\partial z}\mathbf{n}. \qquad (9)$$

As a consequence of [9], the above assumption on P_K and the assumption that the interface and wall pressure terms in [6] are free from derivatives and can be rejected to the R.H.S. *are not compatible.* When, as it is often the case, both assumptions are made notwithstanding, the model is not consistent. It is then found that the set of partial differential balance equations is *hyperbolic* with [6] and is *not* hyperbolic with [7]. This brings out positively that the problem of the nature of the balance equation set is primarily a problem of mathematical form of the constitutive terms (interfacial and wall terms) as stated several times by the author (for instance in Bouré 1975, 1976), but certainly not a problem related to the choice between [6] and [7].

Consideration of the interfacial jump condition

$$\Sigma_K\left\{\int_I[\rho_K(\mathbf{V}_K - \mathbf{V}_I)\cdot\mathbf{n}_K]\mathbf{V}_K\,\mathrm{d}\Sigma + \int_I P_K\mathbf{n}_K\,\mathrm{d}\Sigma - \int_I \bar{\bar{\tau}}_K\cdot\mathbf{n}_K\,\mathrm{d}\Sigma\right\} + \text{surface tension terms} = 0,$$

which contains a term $\int_I (P_G - P_L)\mathbf{n}_G\,\mathrm{d}\Sigma$ (where the subscripts G and L refer respectively to the gas and liquid phases), leads to the following recommendations (Bouré 1978):

In advanced models, in which P_G may be different from P_L, use of form [6] should be preferred.

In all other cases, form [7], which is more compact, should be preferred.

4. THE TWO FORMS OF THE ENERGY EQUATION FOR ONE PHASE

The *local-instantaneous* energy balance may be written

$$\frac{\partial}{\partial t}\left[\rho_K\left(E_K + \frac{\mathbf{V}_K^2}{2}\right)\right] + \nabla\cdot\left[\rho_K\left(E_K + \frac{\mathbf{V}_K^2}{2}\right)\mathbf{V}_K - \bar{\bar{T}}_K.\mathbf{V}_K + \mathbf{J}_K\right] - \rho_K\mathbf{g}\cdot\mathbf{V}_K = 0, \qquad [10]$$

E being the internal energy per unit of mass and J the superficial heat flux.

Introducing in [10] the enthalpy H per unit of mass through the expression

$$\rho_K E_K = \rho_K H_K - P_K \tag{11}$$

and using [2] results in an equation

$$\frac{\partial}{\partial t}\left[\rho_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\right]-\frac{\partial P_K}{\partial t}+\nabla\cdot\left[\rho_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\mathbf{V}_K-\bar{\bar{\tau}}_K\cdot\mathbf{V}_K+\mathbf{J}_K\right]-\rho_K\mathbf{g}\cdot\mathbf{V}_K=0, \tag{12}$$

which is strictly equivalent to [10].

Following the procedure detailed in section 2 for the momentum equation enables the averaged energy equation to be written either:

$$\begin{aligned}
&\frac{\partial}{\partial t}\left[\mathscr{V}\alpha_K\left\langle\rho_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\right\rangle\right]+\frac{\partial}{\partial z}\left[\mathscr{V}\alpha_K\left\langle\rho_K W_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\right\rangle\right]\\
&-\frac{\partial}{\partial t}[\mathscr{V}\alpha_K\langle P_K\rangle]-\frac{\partial}{\partial z}[\mathscr{V}\alpha_K\langle(\bar{\bar{\tau}}_K\cdot\mathbf{n})\cdot\mathbf{V}_K\rangle]+\frac{\partial}{\partial z}[\mathscr{V}\alpha_K\langle\mathbf{J}_K\cdot\mathbf{n}\rangle]\\
&+\int_I\rho_K(\mathbf{V}_K-\mathbf{V}_I)\cdot\mathbf{n}_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\mathrm{d}\Sigma+\int_I P_K\mathbf{V}_I\cdot\mathbf{n}_K\,\mathrm{d}\Sigma-\int_I(\bar{\bar{\tau}}_K\cdot\mathbf{n}_K)\cdot\mathbf{V}_K\,\mathrm{d}\Sigma\\
&+\int_I\mathbf{J}_K\cdot\mathbf{n}_K\,\mathrm{d}\Sigma+\int_{C_K}\rho_K(\mathbf{V}_K-\mathbf{V}_C)\cdot\mathbf{n}_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\mathrm{d}\Sigma+\int_{C_K}P_K\mathbf{V}_C\cdot\mathbf{n}_K\,\mathrm{d}\Sigma\\
&-\int_{C_K}(\bar{\bar{\tau}}_K\cdot\mathbf{n}_K)\cdot\mathbf{V}_K\,\mathrm{d}\Sigma+\int_{C_K}\mathbf{J}_K\cdot\mathbf{n}_K\,\mathrm{d}\Sigma-\mathscr{V}\alpha_K\langle\rho_K\mathbf{V}_K\rangle\cdot\mathbf{g}=0,
\end{aligned} \tag{13}$$

starting from [10], or

$$\begin{aligned}
&\frac{\partial}{\partial t}\left[\mathscr{V}\alpha_K\left\langle\rho_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\right\rangle\right]+\frac{\partial}{\partial z}\left[\mathscr{V}\alpha_K\left\langle\rho_K W_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\right\rangle\right]\\
&-\mathscr{V}\alpha_K\left\langle\frac{\partial P_K}{\partial t}\right\rangle-\frac{\partial}{\partial z}\left[\mathscr{V}\alpha_K\langle(\bar{\bar{\tau}}_K\cdot\mathbf{n})\mathbf{V}_K\rangle\right]+\frac{\partial}{\partial z}[\mathscr{V}\alpha_K\langle\mathbf{J}_K\cdot\mathbf{n}\rangle]\\
&+\int_I\rho_K(\mathbf{V}_K-\mathbf{V}_I)\cdot\mathbf{n}_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\mathrm{d}\Sigma-\int_I(\bar{\bar{\tau}}_K\cdot\mathbf{n}_K)\cdot\mathbf{V}_K\,\mathrm{d}\Sigma\\
&+\int_I\mathbf{J}_K\cdot\mathbf{n}_K\,\mathrm{d}\Sigma+\int_{C_K}\rho_K(\mathbf{V}_K-\mathbf{V}_C)\cdot\mathbf{n}_K\left(H_K+\frac{\mathbf{V}_K^2}{2}\right)\mathrm{d}\Sigma\\
&-\int_{C_K}(\bar{\bar{\tau}}_K\cdot\mathbf{n}_K)\cdot\mathbf{V}_K\,\mathrm{d}\Sigma+\int_{C_K}\mathbf{J}_K\cdot\mathbf{n}_K\,\mathrm{d}\Sigma-\mathscr{V}\alpha_K\langle\rho_K\mathbf{V}_K\rangle\cdot\mathbf{g}=0,
\end{aligned} \tag{14}$$

starting from [12].

The first wall term is usually omitted in both of these equations, since it is zero whenever there is no mass transfer through the pipe wall. The wall term involving the deviatoric stress tensor is also usually omitted, since it is zero whenever the pipe wall is fixed with respect to the frame of reference. Finally, the last two differential terms are usually neglected.

Equations [13] and [14] are two forms of the energy equation which are strictly equivalent. This can be easily verified by using [4] once more with $f_K = P_K$. The result is

$$\int_{\mathscr{V}_K}\frac{\partial P_K}{\partial t}\,\mathrm{d}\mathscr{V}=\frac{\partial}{\partial t}\int_{\mathscr{V}_K}P_K\,\mathrm{d}\mathscr{V}-\int_I P_K\mathbf{V}_I\cdot\mathbf{n}_K\,\mathrm{d}\Sigma-\int_{C_K}P_K\mathbf{V}_C\cdot\mathbf{n}_K\,\mathrm{d}\Sigma,$$

i.e. with a change of sign,

$$\underbrace{-\mathscr{V}\alpha_K\left\langle\frac{\partial P_K}{\partial t}\right\rangle}_{\text{term of [14]}} = \underbrace{-\frac{\partial}{\partial t}[\mathscr{V}\alpha_K\langle P_K\rangle] + \int_I P_K \mathbf{V}_I \cdot \mathbf{n}_K \, d\Sigma + \int_{C_K} P_K \mathbf{V}_C \cdot \mathbf{n}_K \, d\Sigma}_{\text{terms of [13].}} \qquad [15]$$

A discussion, similar to the discussion of section 3, can be made. In particular, the assumption used to derive [9] transforms [15], after division by Δz, into

$$\lim_{(\Delta z \to 0)} \frac{1}{\Delta z}\left[\int_I P_K \mathbf{V}_I \cdot \mathbf{n}_K \, d\Sigma + \int_{C_K} P_K \mathbf{V}_C \cdot \mathbf{n}_K \, d\Sigma\right] = P_K \frac{\partial(\mathscr{A}\alpha_K)}{\partial t}, \qquad [16]$$

an equation which involves derivatives.

When P_G may be different from P_L, use of form [13] should be preferred. In all other cases, from [14], which is more compact, should be preferred.

5. CONCLUSION

It has been shown that, for the phasic momentum equations as well as for the phasic energy equations, two forms, which are strictly equivalent, can be used in two-phase flow modelling. They are given as [6] and [7] for the momentum equation, [13] and [14] for the energy equation. They are equivalent, provided, of course, they are correctly written. A widespread example of inconsistency has been discussed.

Recommendations for the choice between the above forms have been made.

Finally, it has been shown that occurence of complex characteristic directions is not related to the correct use of the above forms but to assumptions made on the mathematical forms of the interfacial and wall constitutive terms.

Acknowledgements—The above contribution has been made possible thanks to the work done by Dr. J. M. Delhaye about the two-phase flow balance equations.

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